Adaptive control for a class of nonlinear systems and application to hard disk drives

Lei Liu¹, Dong-Juan Li² and Yan-Jun Liu¹

Abstract
In this paper, an adaptive controller is investigated for a class of uncertain single-input-single-output nonlinear systems preceded by an unknown backlash-type hysteresis nonlinearity, which is modeled by a differential equation. Backstepping design procedure is employed to construct the control input and adaptation laws. A new algorithm is designed to reduce the computational burden compared with previous work for unknown backlash-type hysteresis. Based on Lyapunov stability theory, all the signals in the closed-loop system are bounded, and the system output can track the reference signal to a bounded compact set. An example for hard disk drives with hysteresis friction nonlinearity is given to verify the feasibility of the proposed approach.

Keywords
Adaptive control, backstepping technique, backlash-type hysteresis, hard disk drives, uncertain nonlinear systems

Received: 28 June 2012; accepted: 20 August 2012

1. Introduction
In recent years, controller design, stability analysis and observer design for several classes of nonlinear systems have received much attention (Busawon et al., 2001; Ngoc, 2012). Specifically, adaptive design strategies have also attracted many researchers to control uncertain nonlinear systems. For example, Kanellakopoulos et al. (1991) developed a systematic procedure for the design of adaptive regulation and tracking schemes for a class of feedback linearizable nonlinear systems. They require that the nonlinear system be transformable into the so-called parametric-pure feedback form. A summary of nonlinear and adaptive control design was given by Krstic et al. (1995). An adaptive output feedback control approach was studied by Liu and Wang (2012) for a class of uncertain nonlinear systems in the parametric output feedback form. Unlike the previous work on adaptive output feedback control, the problem of ‘explosion of complexity’ of the controller in conventional backstepping design is overcome in this paper by introducing the dynamic surface control technique. The work in Yao and Wang (2012) is focused on an underwater manipulator. To solve its nonlinear dynamics and hydrodynamics when it operates underwater, model reference adaptive control is applied. Based on adaptive inverse control theory, combined with a neural network, a neural network adaptive inverse controller was developed in (Yao et al., 2011) and applied to an electro-hydraulic servo system. Some other significant results were also studied (Chen and Li, 2008; Li, 2008; Li et al., 2008, 2010, 2011, 2012; Yang et al., 2008, 2009, 2011; Tong et al., 2009, 2010; Chen et al., 2011; Zhou et al., 2011). However, these results did not consider the effect of input nonlinearity, such as dead zone, backlash, etc.

Recently, much work has been carried out on adaptive control for uncertain nonlinear systems with input nonlinearities. An adaptive dead zone inverse was...
proposed for the control of systems containing an unknown dead zone (Cho and Bai, 1998). It was shown that the effect of the unknown dead zone on the closed-loop control system can be eliminated asymptotically if both input and output measurements of the dead zone are available. The adaptive control of a class of continuous-time nonlinear dynamic systems preceded by an unknown dead zone was dealt with by Wang et al. (2004). By using a new description of a dead zone and by exploring the properties of this dead-zone model intuitively and mathematically, a robust adaptive control scheme is developed without constructing the dead-zone inverse. Several adaptive control schemes have been designed to handle many nonsmooth nonlinearity characteristics such as hysteresis and backlash (Tao and Kokotovic, 1995; Su et al., 2000, 2005; Zhou et al., 2004, 2007). These schemes are obtained for uncertain nonlinear systems with parametric linearly. Subsequently, an adaptive neural control was investigated for a class of unknown nonlinear systems in pure-feedback form with the generalized Prandtl–Ishlinskii hysteresis input (Ren et al., 2009). To deal with the nonaffine problem in the face of the nonsmooth characteristics of hysteresis, the mean-value theorem is applied successively, and unknown uncertainties are compensated for using the function approximation capability of neural networks. A projective synchronization problem of master–slave chaotic systems subject to both sector nonlinearities are compensated for using the function 

\begin{equation}
\frac{d\omega}{dt} = \alpha \frac{|dv|}{|dt|} (cv - \omega) + \beta \frac{dv}{dt} + d(v)
\end{equation}

where $\alpha$, $c$ and $\beta$ are constants and the slope of the lines $c$ satisfies $c > \beta > 0$. Based on equation (1), equation (2) becomes

\begin{equation}
\omega(t) = cv(t) + d(v)
\end{equation}

where $d(v) = [\omega_0 - c\nu_0]e^{-\sigma v - \nu_0} \text{sgn} v + \int_{\nu_0}^{v} \beta - c \\text{sgn} \varepsilon + e^{-\sigma v} \text{sgn} v$. It is proved that dynamic (2) can model a class of backlash-type hysteresis.

The control objective is to design a backstepping adaptive control law to ensure that all the signals in the closed-loop are uniformly ultimately bounded and the system output $y(t)$ tracks a given reference signal $r(t)$ whose $(n - 1)$ th-order derivatives are unknown but bounded to a small compact set, that is $\lim_{t \to \infty} |x(t) - y(t)| \leq \varepsilon_1$ for an specified bounded $\varepsilon_1$.

3. The adaptive controller designs

Based on the above analysis, the following system description is equivalent to the original system (1)

\begin{equation}
\dot{x}(t) + \sum_{i=1}^{l} a_i Y_i(x(t), \dot{x}(t), \ldots, \dot{x}^{(n-1)}(t)) = y v(t) + d_1(t)
\end{equation}

where $y = bc > 0$ and $d_1(t) = bd(v(t)) + d(t)$ with its bounded $|d_1(t)| \leq D$.硬盘驱动器带有迟滞摩擦非线性。文章提出了控制系统中迟滞和反冲的自适应方法。为了验证所提出方法的有效性，给出了一个应用实例。
Now, rewrite equation (4) in the following forms

\[
\begin{aligned}
\dot{x}_i &= x_{i+1}, i = 1, \ldots, n-1 \\
\dot{x}_n &= -\sum_{i=1}^{l} a_i Y(x_1(t), x_2(t), \ldots, x_{n-1}(t)) \\
+\gamma v(t) + d_1(t) &= a^T Y + \gamma v(t) + d_1(t)
\end{aligned}
\]  

(5)

where \( x_1 = x, x_2 = \ldots, x_n = x^{(n-1)}, a = [-a_1, -a_2, \ldots, -a_l]^T \) and \( Y = [Y_1, Y_2, \ldots, Y_l]^T \).

The following change of coordinates is made

\[
\begin{aligned}
z_1 &= x_1 - y_r \\
z_i &= x_i - y_r^{(i-1)} - \eta_{i-1}, \ i = 2, \ldots, n
\end{aligned}
\]  

(6)

(7)

where \( \eta_{i-1} \) means the virtual control which will be determined in later discussion.

**Step 1:** define \( z_1 = x_1 - y_r \) as in (6) and obtain

\[
\dot{z}_1 = z_2 + \eta_1
\]  

(8)

Then, select the virtual control law \( \eta_1 \) as

\[
\eta_1 = -h_1 z_1
\]  

(9)

where \( h_1 \) is a positive parameter that is designed by the author. From the above two equations

\[
z_1 \dot{z}_1 = -h_1 z_1^2 + z_1 z_2
\]  

(10)

Step 1(i) (\( i = 2, \ldots, n-1 \)) : choose the following virtual control as

\[
\eta_i = -h_i z_i - z_{i-1} + \hat{\eta}_{i-1}(x_1, \ldots, x_{i-1}, y_r, \ldots, y_r^{(i-1)})
\]  

(11)

where \( h_i, i = 2, \ldots, n-1 \) are positive parameters. From equations (7) and (11)

\[
z_i \dot{z}_i = -z_{i-1} z_i - h_i z_i^2 + z_i z_{i+1}
\]  

(12)

**Step n:** from equations (5) and (7)

\[
\dot{z}_n = \gamma v(t) + a^T Y + d_1(t) - y_r^{(n)} - \hat{\eta}_{n-1}
\]  

(13)

Then, select the adaptive control law as

\[
\begin{aligned}
v &= \hat{v}, \quad \hat{v} &= -h_n z_n - z_{n-1} - z_n \hat{\rho}^2 |Y|^2 \\
&\quad - \frac{|\text{sgn}(z_n)\hat{D} + Y_r^{(n)} + \hat{\eta}_{n-1}|}{\hat{\rho}|z_n||Y| + \varepsilon}
\end{aligned}
\]  

(14)

\[
\begin{aligned}
\dot{\hat{v}} &= -\phi \hat{v} - \phi \zeta_n \hat{D} + \hat{\rho} d_1(t) + r|z_n||Y|, \\
\dot{\hat{\rho}} &= -r \hat{\rho} + r|z_n||Y|, \\
\hat{D} &= -\theta \hat{D} + \theta r|z_n|
\end{aligned}
\]  

(15)

where \( h_n, \phi, \theta \) and \( r \) are four positive design parameters, \( \varepsilon \) is a positive constant, \( \hat{v}, \hat{\rho}, \hat{D} \) are estimates of \( v = e - \hat{v}, \hat{\rho}, \hat{D} = D - \hat{D} \). Let \( \hat{e} = e - \hat{v}, \hat{a} = a - \hat{a} \), and \( \hat{D} = D - \hat{D} \). Note the fact that \( \gamma v(t) \) in (13) can be written as

\[
\gamma v(t) = \gamma \hat{e} \hat{v} = \hat{v} - \gamma \hat{e} \hat{v}
\]  

(16)

Based on equations (13)-(15)

\[
\begin{aligned}
\dot{z}_n &= -h_n z_n - z_{n-1} - \text{sgn}(z_n)\hat{D} + d_1(t) \\
&\quad - \gamma \hat{e} \hat{v} + a^T Y - \frac{z_n \hat{\rho}^2 |Y|^2}{\hat{\rho}|z_n||Y| + \varepsilon}
\end{aligned}
\]  

(17)

Chose the Lyapunov function as

\[
V = \sum_{i=1}^{n} \frac{1}{2} z_i^2 + \frac{1}{2} \hat{\rho}^2 + \frac{\gamma}{2\phi} \hat{e}^2 + \frac{1}{2\theta} \hat{D}^2
\]  

(18)

Then, the first differential of (18) is

\[
\dot{V} = \sum_{i=1}^{n} z_i \dot{z}_i + \frac{1}{\phi} \hat{\rho} \dot{\hat{\rho}} + \frac{\gamma}{\phi} \hat{e} \dot{\hat{e}} + \frac{1}{\theta} \hat{D} \dot{\hat{D}}
\]  

(19)

Since \( \dot{\hat{\rho}} = -\hat{\rho}, \dot{\hat{v}} = -\hat{v} \) and \( \dot{\hat{D}} = \hat{D} \), equation (19) can be rewritten as

\[
\dot{V} = \sum_{i=1}^{n} z_i \dot{z}_i - \frac{1}{\phi} \hat{\rho} \dot{\hat{\rho}} - \frac{\gamma}{\phi} \hat{e} \dot{\hat{e}} - \frac{1}{\theta} \hat{D} \dot{\hat{D}}
\]  

(20)

Substituting (12), (13) and (17) into (20) leads to

\[
\dot{V} = -\sum_{i=1}^{n} h_i z_i \dot{z}_i - \frac{1}{\phi} \hat{\rho} \dot{\hat{\rho}} - \frac{\gamma}{\phi} \hat{e} \dot{\hat{e}} - \frac{1}{\theta} \hat{D} \dot{\hat{D}} - \text{sgn}(z_n)\hat{D} + z_n \hat{\rho}^2 |Y|^2
\]  

(21)

Consider the fact that \( |a| \leq \rho \), which gives

\[
\begin{aligned}
&\sum_{i=1}^{n} h_i z_i \dot{z}_i - \frac{1}{\phi} \hat{\rho} \dot{\hat{\rho}} - \frac{\gamma}{\phi} \hat{e} \dot{\hat{e}} - \frac{1}{\theta} \hat{D} \dot{\hat{D}} - \text{sgn}(z_n)\hat{D} + z_n \hat{\rho}^2 |Y|^2 \\
&\leq \sum_{i=1}^{n} h_i z_i |z_i| |Y| - \frac{z_n \hat{\rho}^2 |Y|^2}{\hat{\rho}|z_n||Y| + \varepsilon} \leq \varepsilon + \hat{\rho}|z_n||Y|
\end{aligned}
\]  

(22)

By substituting (15) and (22) into (21), we have

\[
\begin{aligned}
\dot{V} &\leq -\sum_{i=1}^{n} h_i z_i^2 + \hat{\rho} \dot{\hat{\rho}} + \gamma \hat{e} \dot{\hat{e}} + \hat{D} \dot{\hat{D}} - D|z_n| + |z_n| |d_1(t)| + \varepsilon
\end{aligned}
\]  

(23)
Due to the inequalities
\[
\tilde{\rho}\dot{\rho} - \dot{\rho}^2 \leq \frac{1}{2}\rho^2 + \frac{1}{2}\dot{\rho}^2 - \dot{\rho}^2 = -\frac{1}{2}\rho^2 + \frac{1}{2}\dot{\rho}^2 \quad (24)
\]
\[
\tilde{e}\dot{e} \leq -\frac{1}{2}e^2 + \frac{1}{2}e^2, \tilde{D}\dot{D} \leq -\frac{1}{2}D^2 + \frac{1}{2}D^2 \quad (25)
\]
then
\[
\dot{V} \leq \sum_{i=1}^{n} 2h_i \left(-\frac{1}{2}z_i^2\right) + r \left(-\frac{1}{2}\tilde{\rho}^2\right) + \phi \left(-\frac{\gamma}{2\phi} \tilde{\rho}^2\right) + \theta \left(-\frac{1}{2\theta} \tilde{D}^2\right) + \frac{1}{2}e^2 + \frac{1}{2}D^2 + \epsilon
\]
(26)

If we choose \( \delta = \min \{2h_i, r, \frac{\phi}{\gamma}, \theta\} > 0 \) and \( \tau = \frac{1}{2}\rho^2 + \frac{1}{2}e^2 + \frac{1}{2}D^2 + \epsilon > 0 \)
\[
\dot{V} \leq -\delta V + \tau
\]
(27)

Then, based on the above, we have the following stability and performance results.

**Theorem 1**: Consider the closed-loop system (1), by choosing the virtual control (9) and (11), the practical control law (14), and the adaptive control law (15), the following statements hold.

1. There exists a large enough compact set \( \Omega \) so that all the signals in the resulting closed-loop system are uniformly bounded.
2. The transient tracking error performance is given by
\[
\Omega_2 = \{z_i | \lim_{t \to \infty} |z_i(t)| = \sqrt{2\frac{\tau}{\delta}, i = 1, \ldots, n}\}
\]
(28)

**Proof**: Multiply both sides of the inequality by \( e^{\delta t} \)
\[
\frac{d}{dt} (V(t)e^{\delta t}) \leq \tau e^{\delta t}
\]
(29)

Integrating (29) over [0, t] gives
\[
0 \leq V(t) \leq \left[V(0) - \frac{\tau}{\delta}\right]e^{-\delta t} + \frac{\tau}{\delta}
\]
(30)

Based on the fact that both \( \delta \) and \( \tau \) are positive constants, the above inequality implies that
\[
0 \leq V(t) \leq V(0)e^{-\delta t} + \frac{\tau}{\delta}
\]
(31)

By making use of (18), we can obtain
\[
\begin{align*}
\frac{1}{2}\tilde{\rho}^2 &\leq V(t), \\
\frac{1}{2\gamma} \tilde{\rho}^2 &\leq V(t), \\
\frac{\gamma}{2\phi} e^2 &\leq V(t), \\
\frac{1}{2\theta} D^2 &\leq V(t)
\end{align*}
\]
(32)

From (31), we get
\[
\begin{align*}
|z_i| &\leq \sqrt{2\left[V(0)e^{-\delta t} + \frac{\tau}{\delta}\right]}, \\
|\tilde{\rho}| &\leq \sqrt{2\left[V(0)e^{-\delta t} + \frac{\tau}{\delta}\right]}, \\
|\tilde{e}| &\leq \sqrt{2\left[V(0)e^{-\delta t} + \frac{\tau}{\delta}\right]}, \\
|\tilde{D}| &\leq \sqrt{2\left[V(0)e^{-\delta t} + \frac{\tau}{\delta}\right]}
\end{align*}
\]
(33)

Since \( \tilde{\rho} = \rho - \rho, \tilde{e} = e - e \) and \( \tilde{D} = \dot{D} - D \), (34) and (33) become
\[
\begin{align*}
|\tilde{\rho}| &\leq \sqrt{2\left[V(0)e^{-\delta t} + \frac{\tau}{\delta}\right]} + |\rho|, \\
|\tilde{e}| &\leq \sqrt{2\left[V(0)e^{-\delta t} + \frac{\tau}{\delta}\right]} + |e|, \\
|\tilde{D}| &\leq \sqrt{2\left[V(0)e^{-\delta t} + \frac{\tau}{\delta}\right]} + |D|
\end{align*}
\]
(35)

Therefore, from (33) and (34), it can be concluded that all the signals in the closed-loop system are uniformly bounded.

From (30) and (32)
\[
|z_i| \leq \sqrt{2\left[V(0) - \frac{\tau}{\delta}\right]e^{-\delta t} + 2\frac{\tau}{\delta}}
\]
(36)

If \( V(0) = \tau / \delta \), then \( |z_i| \leq \sqrt{2\tau / \delta} \). If \( V(0) \neq \tau / \delta \), then given any \( \varphi_2 > 2\sqrt{\tau / \delta} \), there exists a \( T_z \), that for any \( t > T_z \), \( |z_i| \leq \varphi_2 \). Specifically, if \( \varphi_2 = \sqrt{2(V(0) - \tau / \delta)e^{-\delta t} + 2\tau / \delta} \), \( (V(0) \neq \tau / \delta) \) and take \( T_z = -(1 / \delta) \ln\left[\varphi_2^2 - (2\tau / \delta)(2V(0) - (\tau / \delta))\right] \), we have \( \lim_{t \to \infty} |z_i(t)| = \sqrt{2\tau / \delta} \), \( i = 1, \ldots, n \). This completes the proof.

**4. Simulation examples**

Consider the following voice coil motor (VCM) actuator nonlinear system described as
\[
m\ddot{\theta} + h_f(\dot{\theta}, \dot{\theta}) + d(t) = u
\]
(37)
**Figure 1.** The output (solid line) and the reference signal (dashed line).

**Figure 2.** The state $x_2$. 
mass; $\theta$, $\dot{\theta}$ and $\ddot{\theta}$ are the position, velocity and acceleration of the VCM actor read/write (R/W) head tip, respectively; $d(t)$ is the external disturbance; $u$ is the control input; and $h_f(\theta, \dot{\theta})$ is the bearing hysteresis friction of actuator pivot, which is represented as a LuGre friction model consisting of stiffness and viscous friction behavior as (San et al., 2011)

$$ h_f = \mu_1 f + \mu_2 f + \mu_3 \dot{\theta} $$  \hspace{1cm} (38)
where \( f \) is an unmeasured internal state of the friction model; \( \mu_1, \mu_2 \) and \( \mu_3 \) are the hysteresis friction force parameters that can be physically explained as the stiffness of the bristles, damping coefficient and viscous coefficient; \( f_c, f_s \) and \( \dot{x}_s \) are the Coulomb friction, static friction and Stribeck velocity, respectively; and the nonlinear friction characteristic function \( \alpha(\dot{\theta}) \) is a bounded positive function that can be chosen to describe different friction effects.

The control object is to ensure that the position of the VCM actuator R/W head tip \( \theta \) follows the specified desired trajectory \( \hat{\theta} \) to a small neighborhood of zero and all the signals of the closed-loop system are bounded. The desired reference signal in this simulation is \( \hat{\theta}_d = 0.5 \sin(2t) \).

According to the proposed control strategy, the control approach for applying to (37) is summarized in the following. Select the virtual control as \( \eta_1 = -h_1z_1 \) and actual control

\[
\dot{u} = \hat{\theta} \left( -h_2z_2 - z_1 - \frac{z_2^2\ddot{\theta}^2}{\rho |z_2|} Y + e - \text{sgn}(z_2) \hat{D} + y_r^{(2)} + \eta_1 \right).
\]

The adaptive law for \( \dot{\hat{\theta}}, \dot{\rho} \) and \( \hat{D} \) is chosen as

\[
\dot{\hat{\theta}} = -\phi \hat{\theta}, \dot{\rho} = -r \dot{\hat{\theta}}, \dot{\hat{D}} = -\theta \hat{D} + \theta |z_2|
\]

(41)

where \( z_2 = \hat{\theta} - \dot{y}_r - \eta_1 \).

The designed parameters of the proposed control approach are chosen \( \theta = 0.5, r = 0.5, \phi = 0.1, h_1 = 10, h_2 = 5, e = 0.01, \mu_1 = 10^5, \mu_2 = 10^{10}, \mu_3 = 0.4, f_c = 1, f_s = 0.1, x_s = 0.001, m = 1 \), with initial parameters \( \hat{\theta} = 0.1, \rho = 0.1, \hat{D} = 0.1 \) and \( x(0) = -0.5 \).

There has been an increase in demand for smaller hard disk drives with increasingly large data storage capacity. Figures 1 and 2 show the trajectories of the system output and the reference signal. It can be observed that a good tracking performance is achieved. Figure 3 is the output result of the hysteresis nonlinear. Figure 4 shows the adaptation laws.

5. Conclusion

In this paper, an adaptive control approach has been presented for a class of uncertain SISO nonlinear systems with unknown backlash-type hysteresis. A backstepping design procedure is employed to construct the control input and adaptation laws. Compared with the results of most work on unknown backlash-type hysteresis, this paper present a new algorithm to reduce the computational burden. Unlike some other control algorithms, the presented algorithm does not require the model parameter intervals to be known. Based on Lyapunov stability theory, all the signals in the closed-loop system are bounded, and a good tracking performance is achieved. In the simulation examples, a pivot friction compensation method by use of adaptive backstepping control was proposed. It has been shown that the proposed friction compensator can mitigate the hysteresis friction nonlinearity very well.

Funding

This work was supported by the Natural Science Foundation of China (grant no. 61104017) and the Program for Liaoning Excellent Talents in University (grant no. LJQ2011064).

References


