Multiobjective dynamic topology optimization of truss with interval parameters based on interval possibility degree

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Abstract
Static and dynamic multiobjective topology optimization of trusses with interval parameters is investigated. The uncertain parameters of the trusses are described by an interval model. The multiobjective topology optimization model of trusses with interval parameters is constructed. On the basis of Taylor expansion and natural interval extension, the stress and displacement response intervals under static loads and inherent natural frequency interval of truss are deduced. The non-deterministic optimization problem is transformed into a deterministic programming problem by minimizing maximum standards and the concept of interval possibility degree. The Pareto CMOPGA (genetic algorithm for constrained multiobjective optimization problem) embedding structural stability examination on the basis of ranking is adopted to solve the constrained multiobjective optimization problem. Two numerical examples show that the proposed method is effective and reasonable.

Keywords
Genetic algorithm, interval parameters, interval possibility degree, multiobjective, topology optimization, truss

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1. Introduction
Structural dynamic optimization develops from size optimization to shape optimization and topology optimization. It is recognized that topology optimization can greatly improve the design. Some algorithms are put forward for structural dynamic optimization. Xu et al. (2003) developed a topology group concept for trusses with nodal mass and first frequency constraint, where the traditional ground structural method cannot deal with such an optimization problem. Further, because it is very difficult to find optimum designs for structures with multiple eigenvalues by using conventional approaches of optimality criteria method or mathematical programming, a semi-definite programming algorithm is presented for topology optimization of trusses with multiple eigenvalue constraint by Ohsaki et al. (1999). Pedersen and Nielsen (2003) simultaneously carried out size optimization, shape optimization and topology optimization of practical trusses with constraints on eigenfrequencies, displacements, stresses, and buckling, where topology optimization is only considered in the sense that bars of a minimum cross-sectional area will have a negligible influence on the performance of the structure. Fonseca and Bainum (2004) investigated the integrated structural/control optimization of a large space structure with a robot arm subject to the gravity-gradient torque through a semi-analytical approach. Pan and Wang (2006) adopted adaptive genetic algorithm (AGA) for topology optimization of a truss structure with frequency

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domain excitations, where the optimization constraints included fundamental frequency, displacement responses under force excitations and acceleration responses under foundation acceleration excitations. Bai et al. (2009) considered semidefinite programming (SDP) formulations of certain truss topology optimization problems, where a lower bound is imposed on the fundamental frequency of vibration of the truss structure. An integrated design technique to carry out simultaneous topology, shape and sizing optimization of a three-dimensional truss structure, where the design objectives including mass, compliance, natural frequencies, frequency response function (FRF), and force transmissibility (FT) (Noilublao and Bureerat, 2011) was carried out. Chiba et al. (2010) formulated the design of these absorbers as a dynamic optimization problem in which the objective function is the total energy of the uncontrolled structure by optimizing the locations, masses, stiffnesses, and damping coefficients of these absorbers. Obviously, for traditional dynamic optimization of structure, the system parameters are always treated as deterministic.

However, for many practical engineering problems, structural parameters are uncertain, for example there may be inaccuracy of measurement errors of installation and errors of manufacture, etc. Actually, the concept of uncertainty plays an important role in the investigation of various engineering problems. The most common approach to uncertain problems is to model the structural parameters as random/fuzzy variables or fields. In stochastic programming, all information about structural parameters is provided by the joint probability density function (or distribution function) of structural parameters. Unfortunately, probabilistic modeling is not a good way at all times to describe the uncertainty, and the uncertainty is not simply equivalent to the randomness. Indeed, probabilistic approaches are not able to validate the assumptions made regarding the joint probability densities of random variables or functions involved. In fuzzy programming, the constraint functions and the objective function are viewed as fuzzy sets and their membership functions also need to be known. In these two kinds of methods, the membership function and probability distribution play important roles. But, for many practical structures, it is difficult and costly to specify a precise probability distribution or membership functions for an uncertain parameter, and it restricts their wide application to practice. In the past two decades, the interval analysis method has been developed to deal with uncertain problems where the bounds of the uncertain parameters only are required, and it is not necessary to know their probability distributions or membership functions. Because less uncertainty information is needed, this kind of method exhibits an attractive prospect for engineering application. Chen et al. (2004) proposed an interval optimization method to solve the uncertain problems of vibration systems, where the structural characteristics are assumed to be expressed as interval parameters, and derived the interval optimization method by combining the interval extension of function with the first-order Taylor expansions of the functions. Qiu and Wang (2010) studied the anti-optimization problem of structures with uncertain design variables by combining the conventional optimization and interval analysis, and illustrated the feasibility and superiority of the non-probabilistic optimization method in comparison with the conventional and probabilistic optimization methods.

Li et al. (2011) suggested an efficient optimization method named NIO-SLP for uncertain structures by using the non-probabilistic interval model to quantify the uncertainty.

Furthermore, in most real-world problems, several goals must be satisfied simultaneously in order to obtain an optimal solution. The multiple objectives are typically conflicting and non-commensurable. For example, we might want to be able to minimize the total weight of a truss while minimizing its maximum deflection and maximizing its maximum allowable stress. The common approach for this sort of problem is to choose one objective (for example, the weight of the structure) and incorporate the other objectives as constraints. This approach has the disadvantage of limiting the choices available to the designer, making the optimization process a rather difficult task. Another common approach is the combination of all the objectives into a single objective function. This technique has the drawback of modeling the original problem in an inadequate manner, generating solutions that will require a further sensitivity analysis to become reasonably useful to the designer. Correspondingly, a heuristic technique inspired by the mechanics of natural selection (the GA) to solve multiobjective optimization.

Cheng and Li (1998) developed a multiobjective optimization problem to locate the Pareto optimal set, where a new concept called Pareto set filter is adopted. At each generation, the points of rank 1 are put into the filter and undergo a nondominated check. In addition, a niche technique is provided to prevent genetic drift in population evolution. This technique sets a replacement rule for reproduction procedures. The use of the genetic algorithm that can generate better trade-off is demonstrated as a reliable numerical optimization tool (Coelloa and Christiansen, 2000). Madeira et al. (2006) developed a GA for multiobjective topology optimization of linear elastic structures in order to evolve an evenly distributed group of solutions to determine the optimum Pareto set for a given problem.
In this paper, a multiobjective programming scheme for dynamic topology optimization for truss with interval parameters is investigated, where structural mass and maximal nodal displacement are two objective functions and the constraints are imposed on element stress and first natural frequency. The paper is organized as follows: in Section 2, some interval static and dynamic performances are obtained by interval analysis method. Section 3 proposes a new construction method for the evaluation of interval possibility degree in order to compare with different interval numbers. An interval multiobjective optimization problem is presented in Section 4, using the treatment of the uncertain objective functions and the uncertain constraint functions, i.e., introducing a risk index and an interval possibility degree, a non-deterministic multiobjective optimization problem is transformed into a deterministic one. Furthermore, multiobjective GAs embedded with stability examinations of structural topology is used. Next in Section 5, the proposed method is applied for a 15-bar plane truss and a 36-bar space truss with uncertain geometry properties, in which the layout and the means of cross-section areas are optimized to obtain optimal total weight and maximum displacement of the node (both to be minimized). Finally, some meaningful conclusions are drawn in Section 6.

2. Static response and dynamic characteristic analysis for trusses with interval parameters

2.1. Interval analysis method

For dynamic topology optimization of truss, some structural parameters have errors or uncertainties caused by manufacture, installation, computation or measurements. Therefore, it is very important to predict the deviations resulting from the abovementioned uncertainties for structural design. In interval mathematics, the errors or uncertainties are always denoted by intervals. Before we deal with the interval optimization problems, it is necessary to introduce some results of interval analysis.

Let \( \mathbf{x} = (a_i)_{m \times 1} = (a_1, a_2, \ldots, a_m)^T \) be a parameter vector with bounded errors or uncertainties, where the parameter vector lie in intervals as

\[
\underbar{a} \leq \mathbf{x} \leq \overbar{a} \quad \text{or} \quad a_i \in [a_i^L, a_i^U], \quad i = 1, 2, \ldots, m
\]  

where \( \mathbf{a} = (a_i^L) \) and \( \mathbf{b} = (a_i^U) \) are the lower and upper bound vectors of structural parameter vector \( \mathbf{x} \), respectively. \( m \) is the number of interval parameters. In terms of interval matrix notation in interval analysis, the inequality conditions (1) may be written as

\[
\mathbf{x} \in \mathbf{x}' \quad \text{or} \quad \alpha_i \in \alpha_i^L, \quad i = 1, 2, \ldots, m
\]  

where

\[
\alpha_i^L = [a_i^L, a_i^U]
\]  

\( \alpha_i^L \) is the interval parameter vector; \( \alpha_i^L, \alpha_i^R \) are the components of the interval vector \( \alpha_i^L \). Correspondingly, the mean and uncertainty of the interval \( \alpha_i^L \) are

\[
\alpha_i^L = \frac{\alpha_i^L + \alpha_i^R}{2}, \; \Delta \alpha_i = \frac{\alpha_i^L - \alpha_i^R}{2}
\]  

Provided that the uncertainty levels, namely the intervals of uncertain parameters, are relatively small, the objective functions or the constraint functions can be approximately calculated in the uncertainty field through the first-order Taylor expansion

\[
y(x) = y(x^L + \delta x) \approx y(x^L) + \sum_{i=1}^{m} \frac{\partial y(x^L)}{\partial \alpha_i} \delta \alpha_i
\]  

Considering that \( \delta x \) belongs to an interval vector, the interval of the function \( y \) can be obtained through a natural interval extension in interval mathematics

\[
y'(x) = y(x^L) + \sum_{i=1}^{m} \frac{\partial y(x^L)}{\partial \alpha_i} \Delta \alpha_i
\]  

Then the lower bound \( y^L(x) \) and the upper bound \( y^U(x) \) of the function can be obtained explicitly

\[
y^L(x) = \min_x y(x) = y(x^L) - \sum_{i=1}^{m} \left| \frac{\partial y(x^L)}{\partial \alpha_i} \right| \Delta \alpha_i
\]

\[
y^U(x) = \max_x y(x) = y(x^L) + \sum_{i=1}^{m} \left| \frac{\partial y(x^L)}{\partial \alpha_i} \right| \Delta \alpha_i
\]

where the superscripts \( L \) and \( U \) denote lower and upper bounds of the function \( y \), respectively.

2.2. Interval static response

The relationship between the static displacements and the static loads is

\[
K(\alpha)q(\alpha) = F(\alpha)
\]  

where \( K(\alpha) \), \( q(\alpha) \) and \( F(\alpha) \) are system stiffness matrix, displacement vector and static load vector, respectively.
Then we obtain
\[ q' = \frac{\partial q}{\partial \alpha_i} = K^{-1} \left( \frac{\partial F}{\partial \alpha_i} - \frac{\partial K}{\partial \alpha_i} q \right) \]  

(10)

Using Taylor expansion to expand \( q(\mathbf{x}') \) about the mid-vector \( \mathbf{x}' \) of the interval vector \( \mathbf{x}' \) and neglecting the higher order terms, we can have
\[ q(\mathbf{x}') = q(\mathbf{x}') + \sum_i \frac{\partial q(\mathbf{x}')}{\partial \alpha_i} (\alpha_i' - \alpha_i) \]  

(11)

Based on equation (10) and equation (11), we have the interval displacement for the \( h \)th degree of freedom as follows
\[ q_i(\mathbf{x}') = q_i(\mathbf{x}') + \sum_i q_i(\mathbf{x}') (\alpha_i' - \alpha_i) + \sum_i q_i(\mathbf{x}') \Delta \alpha_i \equiv q_i(\mathbf{x}') \]  

(12)

Further, the stress for the \( h \)th element can be expressed as
\[ \sigma_h(\mathbf{x}) = S(\mathbf{x}) T_c q(\mathbf{x}) \]  

(13)

where \( S(\mathbf{x}) \) and \( T_c \) are element stress matrix and element displacement selection matrix, respectively.

Thus the interval stress for the \( h \)th element can be written as
\[ \sigma_h(\mathbf{x}') = \left[ \sigma_h(\mathbf{x}') - \sum_i \left| \frac{\partial \sigma_h(\mathbf{x}')}{\partial \alpha_i} \right| \Delta \alpha_i, \sigma_h(\mathbf{x}') \right] + \sum_i \left| \frac{\partial \sigma_h(\mathbf{x}')}{\partial \alpha_i} \right| \Delta \alpha_i \]  

(14)

2.3. Interval natural frequency

The eigenvalue problem for structural vibration is
\[ K(\mathbf{x}) \varphi_k(\mathbf{x}) = \lambda_k(\mathbf{x}) M(\mathbf{x}) \varphi_k(\mathbf{x}) \]  

(15)

where \( M(\mathbf{x}) \) is system mass matrix; \( \lambda_k \) and \( \varphi_k \) are the \( k \)th eigenvalue and the corresponding eigenvector, respectively.

Then the \( k \)th natural frequency can be obtained as
\[ f_k(\mathbf{x}) = \frac{\sqrt{\lambda_k(\mathbf{x})}}{2\pi} \]  

(16)

Correspondingly, the \( k \)th interval natural frequency can be computed as
\[ f_k(\mathbf{x}') = f_k(\mathbf{x}) \left[ f_k(\mathbf{x}) + \sum_i \left| \frac{\partial f_k(\mathbf{x})}{\partial \alpha_i} \right| \Delta \alpha_i, f_k(\mathbf{x}) \right] + \sum_i \left| \frac{\partial f_k(\mathbf{x})}{\partial \alpha_i} \right| \Delta \alpha_i \]  

(17)

Further, the stress for the \( h \)th element can be written as
\[ \sigma_h(\mathbf{x}') = \left[ \sigma_h(\mathbf{x}') - \sum_i \left| \frac{\partial \sigma_h(\mathbf{x}')}{\partial \alpha_i} \right| \Delta \alpha_i, \sigma_h(\mathbf{x}') \right] + \sum_i \left| \frac{\partial \sigma_h(\mathbf{x}')}{\partial \alpha_i} \right| \Delta \alpha_i \]  

(14)

3. Evaluation of interval possibility degree

The possibility degree of the interval number represents a certain degree that one interval number is larger or smaller than another. Similar to the comparison of interval numbers of Jiang et al. (2008), we extended the probability method into the comparison of interval numbers, and proposed a new construction method for the possibility degree \( P_{a \geq b} \) and \( P_{b \geq a} \) based on three relations of interval numbers \( a \) and \( b \) as shown in Figure 1:

\[ P_{a \geq b} = \begin{cases} 1 & a \geq b \\ \frac{(a - b) - (b - a)}{a} & \frac{a - b}{a} + \frac{a - b}{b} \leq \frac{a - b}{a} \leq \frac{a - b}{b} \\ \frac{b}{a} & \frac{b}{a} + \frac{b}{b} \leq \frac{b}{a} \leq \frac{b}{b} \end{cases} \]  

(19)

\[ P_{b \geq a} = \begin{cases} 1 & b \geq a \\ \frac{(b - a) - (a - b)}{a} & \frac{a - b}{a} + \frac{a - b}{b} \leq \frac{a - b}{a} \leq \frac{a - b}{b} \\ \frac{a}{b} & \frac{a}{b} + \frac{a}{b} \leq \frac{a}{b} \leq \frac{a}{b} \end{cases} \]  

(20)

Here interval numbers \( a \) and \( b \) are regarded as random variables \( \tilde{a} \) and \( \tilde{b} \) with uniform distributions in their intervals. The probability for random variable \( \tilde{a} \) larger or smaller than \( \tilde{b} \) is regarded as \( P_{a \geq b} \) or \( P_{b \geq a} \).
Using equations (19) and (20) to calculate the possibility degree of the same two interval numbers will make the following treatment of the inequality constraints inconvenient. Figure 2 lists all of the possible relations between $a$ and $b$ and based on these relations a modified possibility degree is suggested using the above probability method:

$$P_{a \geq b} = \begin{cases} 
1 & a \geq \bar{b} \\
\frac{\bar{a} - \bar{b}}{\bar{a} - \bar{a}} + \frac{\bar{b} - a}{\bar{a} - \bar{a}} & a \leq \bar{b} \leq \bar{a} \\
\frac{(\bar{a} - \bar{b}) - (\bar{b} - a)}{\bar{a} - \bar{a}} & a \leq \bar{b} \leq \bar{a} \\
\frac{\bar{b} - \bar{a}}{\bar{b} - \bar{a}} & a \leq \bar{a} \leq \bar{b} \\
\frac{(a - b) - (\bar{b} - \bar{a})}{\bar{b} - \bar{a}} & \bar{b} \leq a \leq \bar{a} \leq \bar{b} \\
-1 & a \leq b 
\end{cases}$$

(21)

When $b$ is degenerated into a real number $\bar{b} = [b, b] = b$, all of the possible relations between $a$ and $\bar{b}$ is shown in Figure 3. Here only $a$ is assumed as a random variable $\bar{a}$ with uniform distribution and the degenerated possibility degree $P_{a \geq b}$ can be written:

$$P_{a \geq b} = \begin{cases} 
1 & a \geq \bar{b} \\
\frac{\bar{a} - \bar{b}}{\bar{a} - \bar{a}} - \frac{\bar{b} - a}{\bar{a} - \bar{a}} & a \leq \bar{b} \leq \bar{a} \\
-1 & a \leq \bar{b} 
\end{cases}$$

(22)

Further, when $a$ and $b$ are degenerated into a real number $\bar{a} = [a, a] = a$ and a real number $\bar{b} = [b, b] = b$, respectively. All of the possible relations between $\bar{a}$ and $\bar{b}$ are shown in Figure 4. The degenerated possibility degree $P_{a \geq b}$ can be written as:

$$P_{a \geq b} = \begin{cases} 
1 & a \geq \bar{b} \\
0 & a = \bar{b} \\
-1 & a \leq \bar{b} 
\end{cases}$$

(23)
From the above equations, it can be found that \( L_{a>b} = \lambda (-1 \leq \lambda \leq 1) \) means that \( a \) is larger than \( b \) with the degree \( \lambda \). \( \lambda = 1 \) implies that \( a \) is always larger than \( b \) and \( \lambda = -1 \) implies that \( a \) is always smaller than \( b \).

4. Multiobjective topology optimization of truss with interval parameters based on interval possibility degree

4.1. Optimization problem statement

A general nonlinear interval number programming (NINP) problem with interval parameters in both of the objective function and constraints can be given as follows:

\[
\begin{align*}
\min J(x, \xi) & = [J_1(x, \xi), J_2(x, \xi), \ldots, J_k(x, \xi)] \\
\text{s.t. } g_i(x, \xi) & \geq (-, \leq) \left[ g_{i}^L, g_{i}^U \right] \quad i = 1, 2, \ldots, l \\
x & \in \Omega^n, \xi = [\xi^L, \xi^U], \xi_j = [\xi_j^L, \xi_j^U] \quad j = 1, 2, \ldots, q
\end{align*}
\]

where \( x \) is an \( n \)-dimensional decision vector and \( \Omega^n \) is its range, \( J \) and \( g_i \) are the objective function vector and the \( i \)th constraint, respectively, and \( l \) is the number of the constraints. \( \xi \) is a \( q \)-dimensional uncertain vector and its components are all interval numbers. \( [\xi_j^L, \xi_j^U] \) denotes an interval number and it represents a bounded set of real numbers between the bounds. \( [g_{i}^L, g_{i}^U] \) denotes the allowable interval number of the \( i \)th constraint. For each specific \( x \), the possible values of the objective function or a constraint will form an interval instead of a real number. Furthermore the objective functions and constraints are uncertain and can be nonlinear, thus the traditional deterministic optimization methods and the linear interval number programming methods cannot be used to solve such an optimization problem. In the following sections, a new method will be suggested to solve the above problem.

4.2. Treatment of the uncertain objective function

To certain \( x \), \( J_k(x, \xi) \) is an interval number, and its corresponding interval is denoted by \( \Psi_k = [J_k^L(x), J_k^U(x)] \), where \( J_k^L(x) \) and \( J_k^U(x) \) can be obtained by

\[
J_k^L(x) = \min_{\xi \in \Xi} J_k(x, \xi), \quad J_k^U(x) = \max_{\xi \in \Xi} J_k(x, \xi)
\]

where \( \xi \in \Xi = \{ \xi | \xi^L \leq \xi \leq \xi^U \} \).

For arbitrary \( \lambda_k \in \Psi_k \), we define the risk coefficient as

\[
\alpha_k = \frac{\lambda_k - J_k^L(x)}{J_k^U(x) - J_k^L(x)}
\]

where \( \alpha_k \) denotes a risk index that the objective function really obtained by the decision vector \( x \) is larger than \( \lambda_k \) owing to the uncertainty of \( \xi \). It is obvious that \( \alpha_k \in [0, 1] \). Especially, when \( \lambda = J_k^L(x), \alpha_k = 0 \), it denotes that the decision maker affirmatively obtains the value of the objective function which is larger than or equals to \( J_k^L(x) \); when \( \lambda = J_k^U(x), \alpha_k = 1 \), it denotes that the decision maker is faced with the maximal risk to obtain the value of the objective function which is larger than or equals to \( J_k^L(x) \).
Then we have
\[
\lambda_k = \alpha_k J_k^L(x) + (1 - \alpha_k) J_k^U(x)
\]  
(27)

Equation (27) denotes that the maximal value of the objective function by \( x \) with the risk coefficient \( \alpha_k \) is \( \lambda_k \). In order to simplify the discussion and implementation, we will assume \( \alpha_k = 1 \) in the following section.

4.3. Treatment of the uncertain constraints

In stochastic optimization, we often make the constraints satisfied with a confidence level and transform the uncertain constraints into deterministic constraints. Similarly, we can make the inequality constraint
\[
g_i(x, \xi) \geq \left[ g_{i0}^L, g_{i0}^U \right]
\]
in equation (24) satisfied with a possibility degree level, and formulate a deterministic inequality:
\[
P_{c \geq d} = \eta_i, \quad c = \left[ g_i^L(x), g_i^U(x) \right], \quad d = \left[ g_{i0}^L, g_{i0}^U \right]
\]  
(28)

where \( P_{c \geq d} \) is the possibility degree of the \( i \)th constraint. \(-1 \leq \eta_i \leq 1\) is a predetermined possibility degree level. \( c \) is the interval of the constraint function at \( x \) and the bounds can be obtained through two deterministic optimization processes:
\[
g_i^L(x) = \min_{\xi \in \Xi} g_i(x, \xi), \quad g_i^U(x) = \max_{\xi \in \Xi} g_i(x, \xi)
\]  
(29)

\( \eta_i \) can be adjusted to control the feasible field of \( x \). When \( \eta_i \) is larger, the inequality constraint equation (29) will be restricted more strictly and the feasible field of \( x \) will become smaller. For the inequality constraint \( g_i(x, \xi) \leq \left[ g_{i0}^L, g_{i0}^U \right] \), it can be changed to \( \left[ g_{i0}^L, g_{i0}^U \right] \geq g_i(x, \xi) \) and treated with the above method. For the equality constraint \( g_i(x, \xi) = \left[ g_{i0}^L, g_{i0}^U \right] \), it can be transformed into the following form:
\[
P_{g_i(x) \leq g_{i0}} = 0, \quad P_{g_i(x) \geq g_{i0}} = 0
\]  
(30)

4.4. Deterministic multiobjective optimization problem

Through the above treatments, the NINP problem equation (24) is transformed into the following deterministic multiobjective programming problem. The multiobjective topology optimization model for truss with stress constraint and frequency constraint based on the interval possibility degree is constructed as
\[
\begin{align*}
\text{find} & \quad A_1, A_2, \ldots, A_n, \beta_1, \beta_2, \ldots, \beta_n \\
\text{min} & \quad W, \quad q_{\text{max}} \\
\text{s.t.} & \quad P_{\beta_1 \geq \beta_1} \geq \eta_i, \quad i = 1, 2, \ldots, n_e \\
& \quad P_{f_i \geq f_i} \geq \eta_f
\end{align*}
\]  
(31)

where \( n_e \) is the number of truss elements and \( n_a \) is the number of nodes; \( A_i \) is the cross-section area of the \( i \)th element; \( \beta_i \) is the topology design variable corresponding to the \( i \)th node and its value is 1 or 0, where 1 denotes that the node exists, or else it is removed; \( W \) and \( q_{\text{max}} \) are the upper bounds of structural mass and the maximal nodal displacement, respectively; \( \sigma_i, [\sigma], \) and \( \eta_f \) are the work stress, the allowed stress and the given stress possibility level of the \( i \)th element; \( f_i, [f_i] \) and \( \eta_f \) are structural first natural frequency, the allowed first natural frequency and the corresponding given possibility level.

4.5. Multiobjective genetic algorithms

Commonly, for constrained multiobjective optimization problems, it is crucial to transform a constrained optimization problem into an unconstrained optimization problem by the penalty function method. Then the comparison of individuals with respect to several objectives may be achieved through the introduction of the concepts of Pareto optimality and dominance.

Generally, multiobjective optimization methods are classified into two broad categories:

Priori articulation of preferences: Combining individual objective functions using pre-decided weight factors into a single utility function and solve the problem as a single objective optimization problem. Weight factors express the relative importance of the objective function in the overall utility measure.

Posteriori articulation of preferences: In this method optimizer first generates a number of non-inferior (a set of equally efficient) solutions and then a final decision is made on any one solution. This approach is often referred to as Pareto optimal approach. As a set of many tradeoff solutions are already available with their cons and pros, the Pareto optimal approach helps in high-level qualitative decisions.

The present paper uses the second approach and derives Pareto-optimal front for dynamic topology optimization of trusses with interval parameters. As genetic algorithms use a population of solutions, it is easier to capture multiple non-inferior solutions (Pareto optimal set) in the final population. A rank-based approach without using the penalty method is used to handle a constrained multiobjective optimization problem. The flowchart diagram of a multiobjective genetic algorithm is illustrated in Figure 5. Some basic ideas of genetic algorithm for constrained multiobjective optimization problem (GACMOP) are as following:

1. Ranking and fitness assignment of the individuals. This is based on the following five principles:

(i) In a given population, meaning individuals are preferred over meaningless individuals;
(ii) Feasible individuals are preferred over unfeasible individuals;
(iii) Individuals near to the feasible domain are preferred over those far from the feasible domain;
(iv) Individuals near to the Pareto front are preferred over those far from the Pareto front;
(v) For unfeasible individuals, principle (iii) is preferred over principle (iv).

Thus ranking and fitness of the individuals are evaluated by the following steps:

a. To rank all meaning individuals of current population based on their objective functions, and assume that the number of all meaning individuals and the rank of $i$th meaning individual are respectively denoted by $P_{op}$ and $R_{oi}$;

b. To rank all meaning individuals of current population based on their degree to violate the constraints, and assume that the rank of $i$th meaning individual is denoted by $R_{ci}$; for feasible individual, $R_{ci} = 0$ or else the smaller is $R_{ci}$, the smaller is the degree to violate the constraint;

c. To calculate the total ranking according to the following equation

$$R_i = \begin{cases} R_{oi} & R_i = 0 \text{ (feasible indivudal)} \\ \frac{R_{oi}}{P_{op}} + \frac{R_{ci} + Pop}{5Pop} & R_i \neq 0 \text{ (infeasible indivudal)} \\ & (meaningless indivudal) \end{cases}$$

(32)

d. To compute the fitness of the individual

$$fit_i = \frac{const}{R_i}$$

(33)

where $const = 1e5$ in the paper.

2. Pareto set filter operator. The Pareto set filter operator works during the whole optimization process. All points assigned rank 1 at each generation are registered in a Pareto set file. This file is dynamically updated by using a filter operator. In this filter, the non-dominated solutions of the current population are compared with those already stored in the file, from the previous generations. A new evaluation is conducted in all points in the filter, according to the following rules:

a. All points in the filter reassigned rank 1, i.e. points identified as non-dominated, are recorded in the Pareto set file. The dominated ones are discarded;
b. If the number of points in the file is inferior to the population size, the new non-dominated points are stored. Otherwise, if the file is full, the most similar points in the Pareto set file are replaced.

The concept introduced to measure the similarity between two points in rule (b) is based on the Euclidean distance between the values of the objective functions. The points with minimum distance relative to the others are removed from the file. Such a procedure maintains an even distribution of the points in the file, which helps to provide the optimal set with the best solutions. At the end of the optimization process, the file itself comprises the Pareto optimal set and constitutes the result of the optimization.

3. Rank penalty for excessive individuals. To maintain an even distribution of solutions along the Pareto optimal front, a rank penalty is applied for excessive individuals when the number of the same individuals of a population exceed a specified number, i.e.,

$$R_{new} = R_{old} + Pop$$

(34)

4. Populations recombine. Combine individuals of both the parent and child generations into one unified population whose member count is 2popsize. Individuals in
this unified population are sorted in accordance with their fitness. The first popsize numbers of individuals with better fitness are treated as the new parent population.

5. Search stopping criterion. In order to stop the sequence of successive population generations, a termination criterion based on the evaluation of the solutions in the Pareto set filter and evolutionary generation is adopted. The optimization process is interrupted when a significant progress is no longer obtained in the solutions in the Pareto set filter and a specified evolutionary generation is reached.

6. Code. The decimal value of each design variable is encoded into a gene represented by binary code. Accordingly, the chromosome constructions of different kinds of design variables are shown in Figure 6. In order to speed up the optimal process, nodal information besides the support nodes and the node on which external loads are imposed is also coded into a chromosome, where a value of 1 assigned to a node corresponds to the presence of a node and a value 0 to a removed node. When a node is removed, the cross-sectional areas of all elements connected to such anode are zeros. Then all corresponding chromosomes are joined together to represent a design variable vector.

7. Structural stability examination. The “structural stability examination” is performed by two steps, i.e., “nodal examination” and “rigid body movement examination”. The “nodal examination” is performed by examining whether there is a node, which is only connected to two members. When a node with no external loads is only connected by two members that do not lie along a straight line, the node must be meaningless. Therefore, when a topology has at least such a node, the topology can then be taken as meaningless and it is unnecessary to do any further optimization analysis. The “rigid body movement examination” is performed by examining whether the stiffness matrix is singular. Although all nodes of a certain topology are meaningful, the corresponding structure can still be a mechanism that results in rigid body movement. When a topology creates rigid body movement, it must be unable to bear some types of external loads. Thus, it is meaningless and need not be analyzed any further.

5. Numerical examples

The multiobjective dynamic topology optimizations for a 15-bar plane truss and a 36-bar space truss are investigated, as shown in Figure 7 and Figure 11, respectively. The elastic modulus, mass density, cross-section area of each element, the allowed stress, and the

Figure 6. Flowchart diagram of multiobjective genetic algorithm.
allowed value of the first natural frequency are uncertain, where there is 3% disturbance to these means. The other parameters are deterministic. The means corresponding to mass density and elastic modulus are 2710 kg/m³ and 72 MPa, respectively. The definition domain of the means of cross-sectional areas of common truss elements is

\[ S = \{80, 125, 180, 259.2, 320, 405, 500, 672.2, 793.8, 1125, 1620, 1280, 2000, 2424, 3125\} \text{ mm}^2. \]

The population size is \( P_s = 200 \), the crossover rate is \( P_c = 0.8 \), the mutation rate is \( P_m = 0.02 \). The termination condition is that the evolutionary generation is 500. The stress constraint demands that the possibility degree be greater than 0.9 where the stress for each member is not greater than the allowed value. The first natural frequency constraint demands that the possibility degree is greater than 0.9, where the first natural frequency is greater than the allowed value.

5.1. 15-Bar plane truss

The mean of the allowed stress is 168 MPa and the mean of the allowed values of the first natural frequency is 100 Hz. Both the topology logic variables corresponding to node 3, 4, 5 and 7 and the cross-section area mean of each element are design variables. The static load condition is that a load of 300 KN is imposed in the negative \( y \)-direction of node 8.

The computed Pareto front is shown as in Figure 8. The Pareto set includes nine best individuals, and their structural characteristics are shown in Table 1. The optimal topologies are shown in Figure 9 and the means of cross-section areas are shown in Table 2. The dynamic topology optimization is executed for the deterministic optimization model by the proposed method in the paper, i.e., the uncertainties of all interval parameters are assumed to be zeros. The Pareto set for the deterministic optimization model is shown in

Figure 7. 15-Bar plane truss.

Figure 8. Pareto front of the non-deterministic optimization model for 15-bar plane truss (the discrepancy ratios: 3%).

Figure 9. Optimal topology layouts of the non-deterministic optimization model for 15-bar plane truss (the discrepancy ratios: 3%). (a) Individual 1, 2, 3; (b) Individual 4; (c) Individual 5, 6, 7, 8, 9.
Figure 10. Pareto front of the deterministic optimization model for 15-bar plane truss (the discrepancy ratios: 0%).

Table 1. Optimal characteristics of nine individuals corresponding to non-deterministic optimization model for 15-bar plane truss (the discrepancy ratios: 3%)

<table>
<thead>
<tr>
<th>Individual</th>
<th>$\sigma_{\text{max}}$ (MPa)</th>
<th>$f_1$ (Hz)</th>
<th>$R_C$</th>
</tr>
</thead>
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<td>(154.33,165.92)</td>
<td>(191.72,212.61)</td>
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<tr>
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<td>5</td>
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<td>9</td>
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</tbody>
</table>

Figure 11. 36-Bar space truss.

5.2. 36-Bar space truss

The means of the allowed stress and the allowed first natural frequency are 173.2 MPa and 50 Hz, respectively. The design variables are the topology logic variables corresponding to nodes 5, 6, 7, 8, 9, 10 and 11 and the mean of cross-section area of each element. The static load condition is that a load of 400 KN is imposed in the y-direction of node 12.

The calculated Pareto front is shown as in Figure 12. The Pareto set includes nine best individuals, whose structural characteristics are shown in Table 4. The optimal topologies are shown in Figure 13 and the midpoints of cross-section areas are shown in Table 5.

Further, the dynamic topology optimization is also executed for the deterministic optimization model, where the uncertainties of all interval parameters are assumed to be zeros. The Pareto set for the deterministic optimization model is shown in Figure 14, and there are eight optimal individuals. The structural characteristics for eight optimal individuals are shown in Table 6 and the constraint index $R_C$ is also calculated for the case with 3% discrepancy ratio.

As can be seen in Figure 12 and Figure 14, there is no absolute optimal solution for two competing indexes that are to minimize structural mass and maximal nodal displacement simultaneously. A variety of non-inferior
solutions that are unable to be compared simply are obtained. Table 4 and Table 6 show that the solution corresponding to less mass and larger nodal displacement for the deterministic optimization problem is most likely unfeasible for the non-deterministic optimization problem. When structural mass is smaller, the stress of the optimal solution for the deterministic optimization problem just satisfies the stress constraint. However, if structural parameters have dispersiveness and lead to stress dispersiveness, the stress possibility degree reduces so that it can’t satisfy the required possibility degree constraint. If the discrepancy ratio increases to a higher value, the optimal mass and the nodal displacement are larger.

### Table 2. Optimal means of cross-section areas of nine individuals in the Pareto set for the non-deterministic optimization model for 15-bar plane truss (the discrepancy ratios: 3%)

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<th>Cross-section area (10^{-6}m^2)</th>
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<th>Individual 3</th>
<th>Individual 4</th>
<th>Individual 5</th>
<th>Individual 6</th>
<th>Individual 7</th>
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<th>Individual 9</th>
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### Table 3. Optimal characteristics of 10 individuals corresponding to the deterministic optimization model for 15-bar plane truss (the discrepancy ratios: 0%)

<table>
<thead>
<tr>
<th>Individual</th>
<th>$\sigma_{\text{max}}$ (MPa)</th>
<th>$f_1$ (Hz)</th>
<th>$R_C$</th>
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<td>165.47</td>
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Figure 13. Optimal topology layouts of non-deterministic optimization model for 36-bar space truss (the discrepancy ratios: 3%).
Table 4. Optimal characteristics of nine individuals corresponding to non-deterministic optimization model for 36-bar space truss (the discrepancy ratios: 3%)

<table>
<thead>
<tr>
<th>Individual</th>
<th>$\sigma_{\text{max}}$ (MPa)</th>
<th>$f_1$ (Hz)</th>
<th>$R_c$</th>
</tr>
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Table 5. Optimal means of cross-section areas of nine individuals in the Pareto set for the non-deterministic optimization model for 36-bar space truss (the discrepancy ratios: 3%)

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<th>Cross-section area ($10^{-6}m^3$)</th>
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<th>Individual 2</th>
<th>Individual 3</th>
<th>Individual 4</th>
<th>Individual 5</th>
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(continued)
6. Conclusions

1. The interval analysis method based on Taylor expansion and natural interval extension of the function is introduced. Furthermore, the interval performances of structures with interval parameters are inferred.

2. The multiobjective topology optimization model for trusses with interval parameters is put forward and is transformed into the deterministic optimization model based on interval possibility degree. The Pareto CMOPGA embedding structural effectivity and stability examination on the basis of ranking is applied to multiobjective topology optimization of trusses with interval parameters.

3. The ideal Pareto front is obtained for two numerical examples and can provide a variety of options for the decision maker. Some unnecessary nodes and bars can be removed by structural topology optimization.

4. In essence, the deterministic optimization problem is a special case of the non-deterministic optimization problem when the discrepancy rate of all interval parameters is zero. When the discrepancy rate is not equal to zero, the parameters can take any value from the low bound to the upper bound. Obviously, the possibility degree constraint of the non-deterministic optimization problem is more stringent than that of the deterministic optimization problem. Therefore, the former optimal solutions are possibly unfeasible for the deterministic optimization problem. Because there are uncertainties in engineering practice and the optimal solutions are different for the deterministic optimization problem and the non-deterministic optimization problem, the optimization problem where uncertain parameters are taken into account is more reasonable.

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References


Table 5. Continued

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Table 6. Optimal characteristics of eight individuals corresponding to the deterministic optimization model for 36-bar space truss (the discrepancy ratios: 0%)

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<th>Individual</th>
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<th>(R_C)</th>
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